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Testing and Validation of the Dynamic Inertia Measurement Method

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IMAC XXXIII
A Conference and Exposition on Structural Dynamics

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Introduction/ Motivation

Mass Properties

- Necessary to understand and control the flight dynamics of the vehicle.
  - mass
  - center of gravity (CG)
  - moments of inertia (MOIs)
  - products of inertia (POIs)
- Methods to determine mass properties:
  - **Analytical models** can also provide mass properties information, but must be sufficiently detailed as a realistic representation of the system to be accurate.
  - **Weight and balance** procedures are usually used to determine mass and CG, while MOI and POI require dynamic testing.
  - **Spin-balance tables** can provide accurate approximations of the CG and MOI, but these become increasingly difficult to use as the size of the object being tested increases.
  - **Pendulum-based methods** can also be used. However, pendulum-based methods require significant amounts of labor, materials, and time, leading to high costs and risk to the vehicle and schedule.
  - **Frequency response function (FRF)** testing has gained interest as an alternative methodology for determining mass properties using a ground vibration test (GVT) setup. Frequency response function testing analyzes the dynamic response of a test article and is often used to identify mode shapes and natural frequencies of objects. The Dynamic Inertia Measurement (DIM) method utilizes FRF information to determine mass properties.[

Dynamic Inertia Measurement (DIM) Method

- The DIM method has been in development at the University of Cincinnati and has shown success on a variety of small scale test articles such as automobile brake rotors, steel blocks, and other custom fixtures from the university.

- Attempts to apply the DIM method to larger test articles, however, have met with limited success.
  - X-38, Initial Iron-Bird Test Article

- The DIM method measures the inertia properties of an object by analyzing the frequency response functions measured during a ground vibration test (GVT).

- The mass properties of an object are determined by measuring all forces and moments acting on a body and the rigid body motion caused by these forces and moments.

Equations

The DIM method uses the rigid body forces, moments, and linear and angular accelerations to calculate the inertia matrix. Equation 1 shows Newton's second law simplified for constant mass which defines the relationship between forces, mass, and linear accelerations

$$\{F\} = [M]\{\ddot{x}\} \tag{1}$$

Equation 2 shows Euler's second law for defining the relationship between moments, moments and products of inertia, and angular accelerations. For this solution, the cross terms were ignored because the test articles are assumed to be rigid to an extent that the vehicle rotation rate terms were small. Note that this assumption would not hold for large, flexible structures.

$$\{N\} = [I]\{\ddot{\theta}\} \tag{2}$$

Applying the small angle assumption to the moment arms and combining the force and moment equations for six degrees of freedom yields the 6x6 mass matrix for full rigid body motion as shown in Equation 3. All forces, moments, and accelerations are measured quantities.

$$\begin{Bmatrix} \mathrm{F_x} \\ \mathrm{F_y} \\ \mathrm{F_z} \\ \mathrm{N_x} \\ \mathrm{N_y} \\ \mathrm{N_z} \end{Bmatrix} = \begin{bmatrix} \mathrm{m} & 0 & 0 & 0 & \mathrm{mZ_{CG}} & -\mathrm{mY_{CG}} \\ 0 & \mathrm{m} & 0 & -\mathrm{mZ_{CG}} & 0 & \mathrm{mX_{CG}} \\ 0 & 0 & \mathrm{m} & \mathrm{mY_{CG}} & -\mathrm{mX_{CG}} & 0 \\ 0 & -\mathrm{mZ_{CG}} & \mathrm{mY_{CG}} & \mathrm{I_{xx}} & -\mathrm{I_{xy}} & -\mathrm{I_{xz}} \\ \mathrm{mZ_{CG}} & 0 & -\mathrm{mX_{CG}} & -\mathrm{I_{xy}} & \mathrm{I_{yy}} & -\mathrm{I_{yz}} \\ -\mathrm{mY_{CG}} & \mathrm{mX_{CG}} & 0 & -\mathrm{I_{xz}} & -\mathrm{I_{yz}} & \mathrm{I_{zz}} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{Bmatrix} \tag{3}$$

DIM Set-Up

- All forces, moments, and accelerations are measured quantities. The forces and moments are measured from DIM-related 6-DOF force sensors and shaker input sensors. The accelerations are measured from typical GVT accelerometers.

- The ten unknown terms in the mass matrix (M) are the mass (m), CG location (X_{CG}, Y_{CG}, Z_{CG}) with respect to some point P, moments of inertia (I_{xx}, I_{yy}, I_{zz}) calculated about P, and products of inertia (I_{xy}, I_{xz}, I_{yz}) calculated about P.

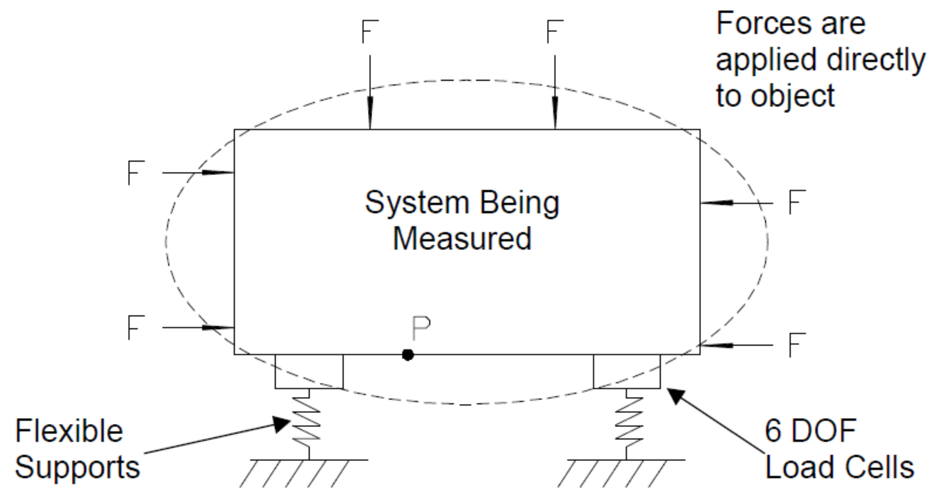


Figure courtesy of :
Witter, M. C., “Rigid Body Inertia Property Estimation Using the Dynamic Inertia Method, Master of Science thesis, Department of Mechanical Engineering, University of Cincinnati, Ohio, 2000.

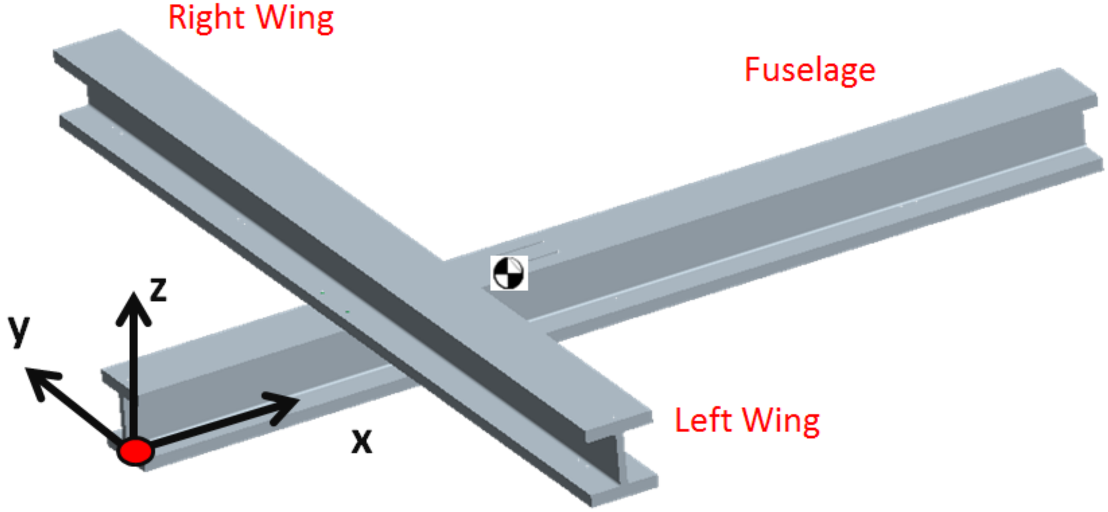
Test Article

- Two 8500-lb 20-foot long, W14x426 steel I-beams were bolted together off-center to model the approximate mass of fighter-type aircraft.
- Since the test article was somewhat visually similar to an aircraft, it was named the “iron bird.”
- The iron bird was intentionally simple in design to ensure high reliability of its analytical mass properties.

Analytical Model

- Pro/ENGINEER® was used to analytically model the iron bird test article and obtain the mass properties. Care was taken to apply as many realistic details to the CAD model as possible including all holes and adding interface attachments in order to ensure accuracy. The simplicity of the iron bird test article design was to ensure the analytical CAD model could be treated as the "truth model." The analytic mass properties of the iron bird from the CAD model are shown below.

Mass, kg	X_{CG}, m	Y_{CG}, m	Z_{CG}, m	I_{xx}, kg*m^2	I_{yy}, kg*m^2	I_{zz}, kg*m^2	I_{xy}, kg*m^2	I_{xz}, (kg*m^2)	I_{yz}, kg*m^2
7716.5	2.3	0.0	0.5	12700.6	17207.2	28503.1	0.0	-1398.8	0.0

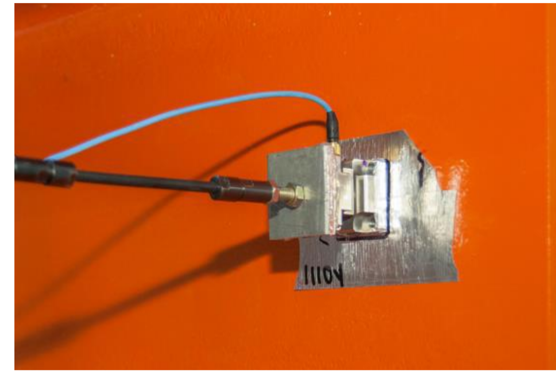


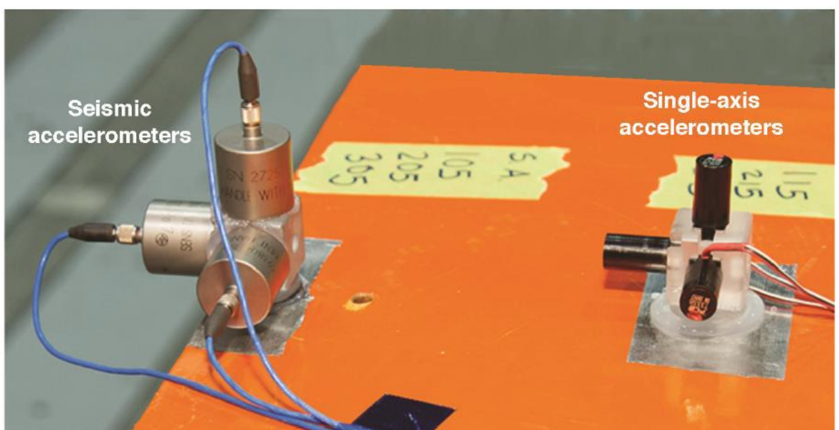
Pendulum Swing Tests

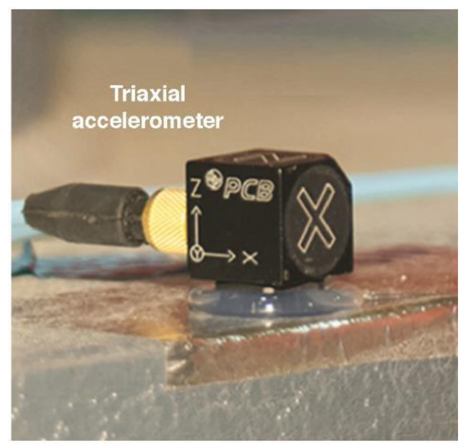
- Classical pendulum equations were used to determine the moments of inertia
- In order to obtain the moments of inertia of the iron bird, all tests also required swinging the fixture by itself in order to subtract out the fixture mass properties from the total combined iron bird and fixture assembly.
- The moments of inertia about the x-axis and y-axis used a compound pendulum setup
- The z-axis MOI uses a bifilar torsional pendulum setup

DIM Equipment

- Soft Support System
  - 3-DOF Load Cells: Soft Supports
- 6-DOF Force Sensors: Reaction Forces
- Shakers: Excitation Input
  - 3-DOF and 1-DOF Force Sensors: Excitation Force
- Accelerometers
- Laser Tracker System



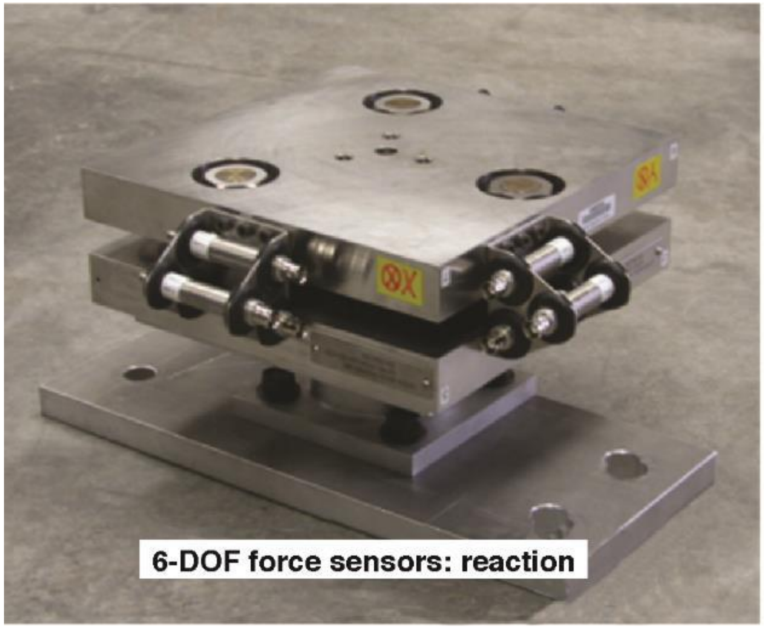


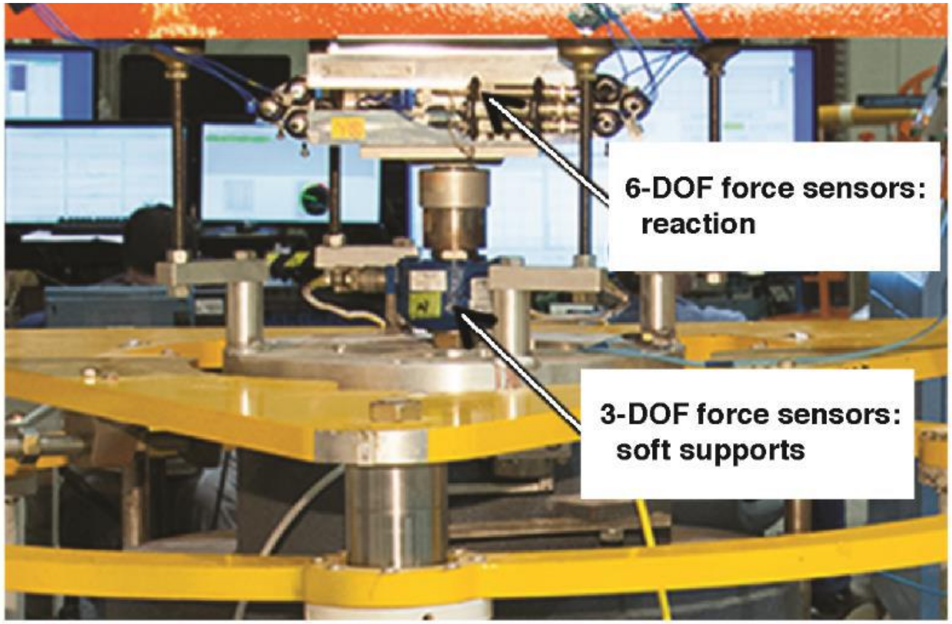


6-DOF Force Sensors

- Three 6-DOF force sensors were custom-made for the NASA AFRC researchers by PCB Piezotronics, Inc. (Depew, New York).

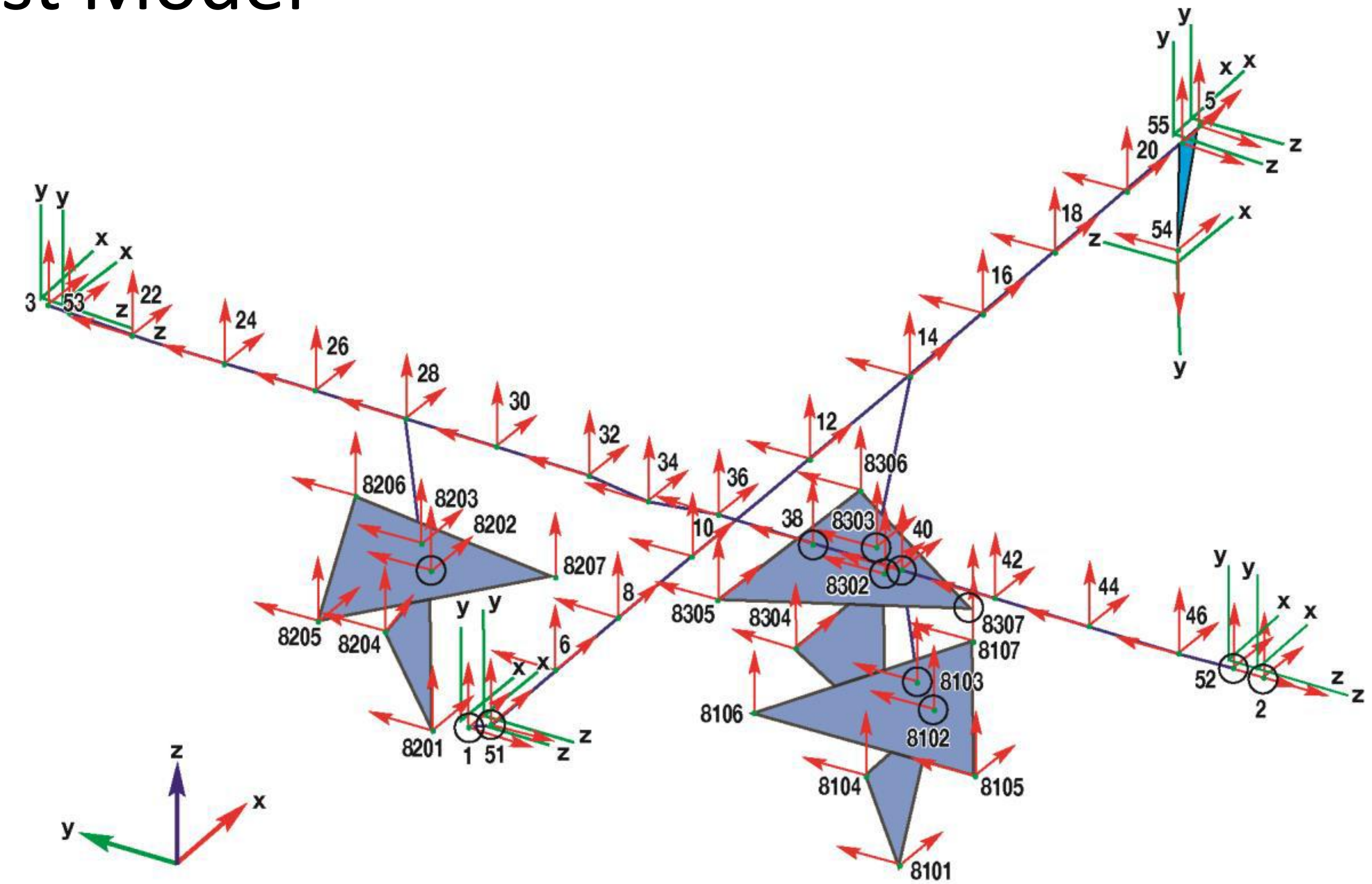
- These unique sensors are an assembly of three 3-DOF piezoelectric dynamic force sensors. The force sensors were placed between the iron bird and the soft-support system.





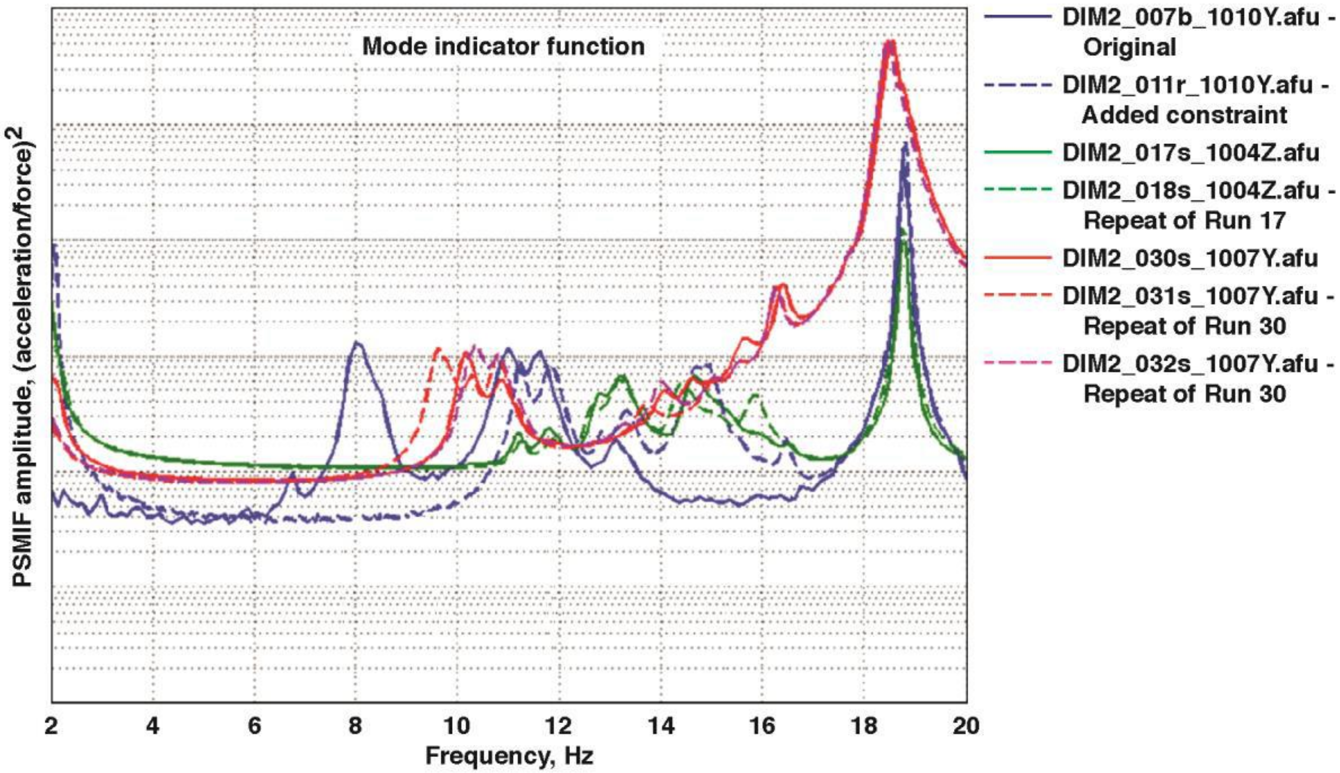
DIM Testing

- Test Model



DIM Testing

- A total of 12 different DIM analysis cases using 54 test runs.
- Restraints were added to the soft supports to create the DIM analysis range of 2-10 Hz.



Mode	Frequency, Hz	Damping, % critical	Description
1	0.72	8.52	Rigid body pitch
2	0.98	6.32	Rigid body roll
3	1.25	3.81	Rigid body yaw
4	1.56	6.21	Rigid body vertical
5	1.80	2.99	Rigid body fore-aft
6	1.99	2.91	Rigid body lateral
7	10.08	1.99	Starboard 60K3S canister X+/Y-

DIM Testing

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- The iron bird DIM testing was conducted at the NASA AFRC Flight Loads Laboratory (FLL) from September 16, 2013 through September 24, 2013. ATA Engineering, Inc. (San Diego, California) was contracted to assist with the iron bird DIM testing and to perform analysis of the data

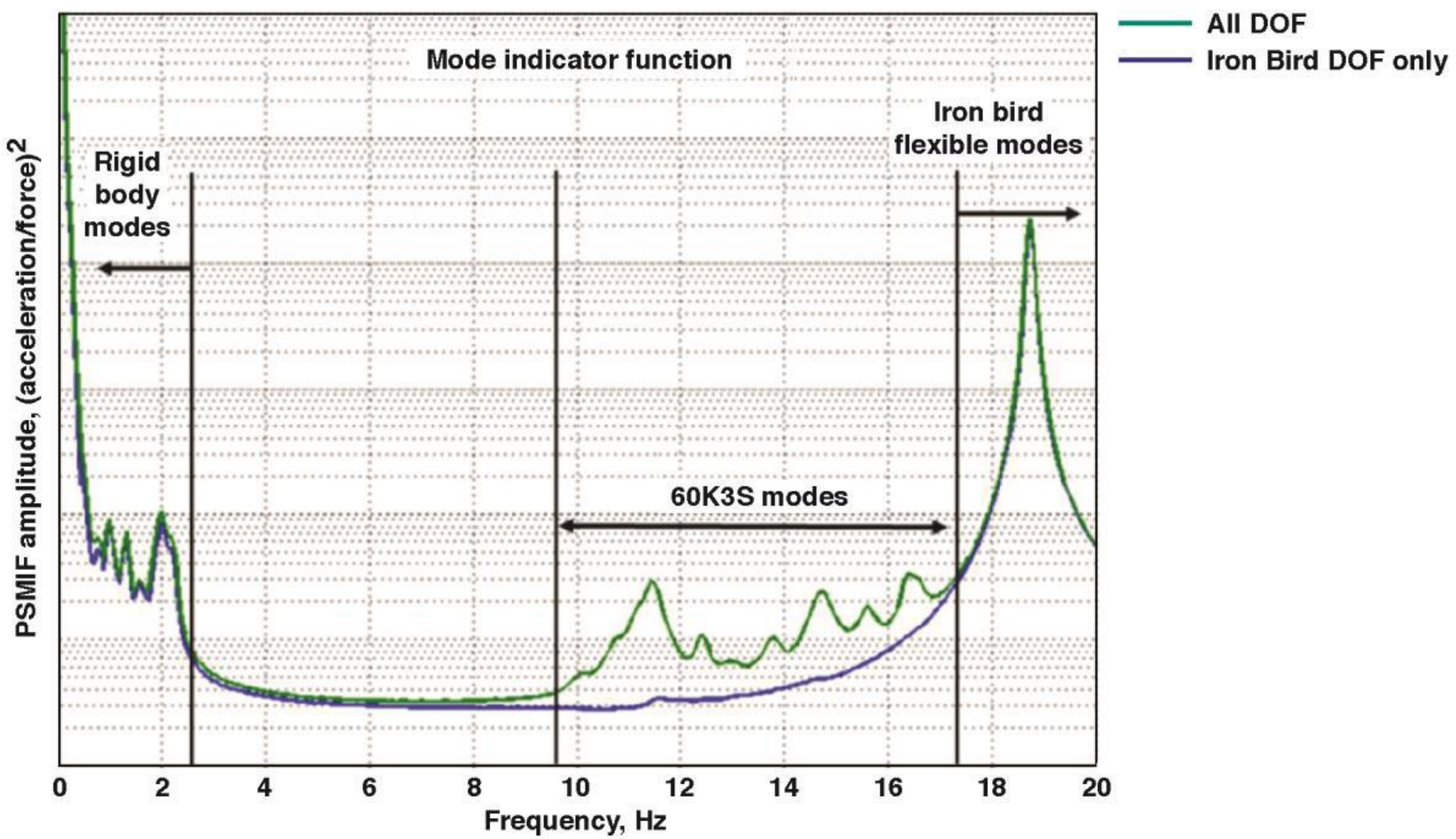
- A total of twelve different DIM analysis cases were conducted through the course of 54 test runs. These runs included check-out, single-shaker, multi-shaker, and quiescent runs.

- Both GVT and seismic accelerometers were used to determine whether higher sensitivity seismic accelerometers are required

Analysis case	Shaker configuration	Shaker excitation	Accelerometers
1	Single	Random (0 to 100 Hz)	Seismic
2	Single	Random (0 to 12 Hz)	Seismic
3	Single	Sine sweep (1 to 20 Hz)	Seismic
4	Single	Random (0 to 100 Hz)	GVT
5	Single	Random (0 to 12 Hz)	GVT
6	Single	Sine sweep (1 to 20 Hz)	GVT
7	Double	Random (0 to 100 Hz)	Seismic
8	Double	Random (0 to 12 Hz)	Seismic
9	Double	Sine sweep (1 to 20 Hz)	Seismic
10	Double	Random (0 to 100 Hz)	GVT
11	Double	Random (0 to 12 Hz)	GVT
12	Double	Sine sweep (1 to 20 Hz)	GVT

DIM Analysis

- PSMIF for 2-shaker random 0-100Hz

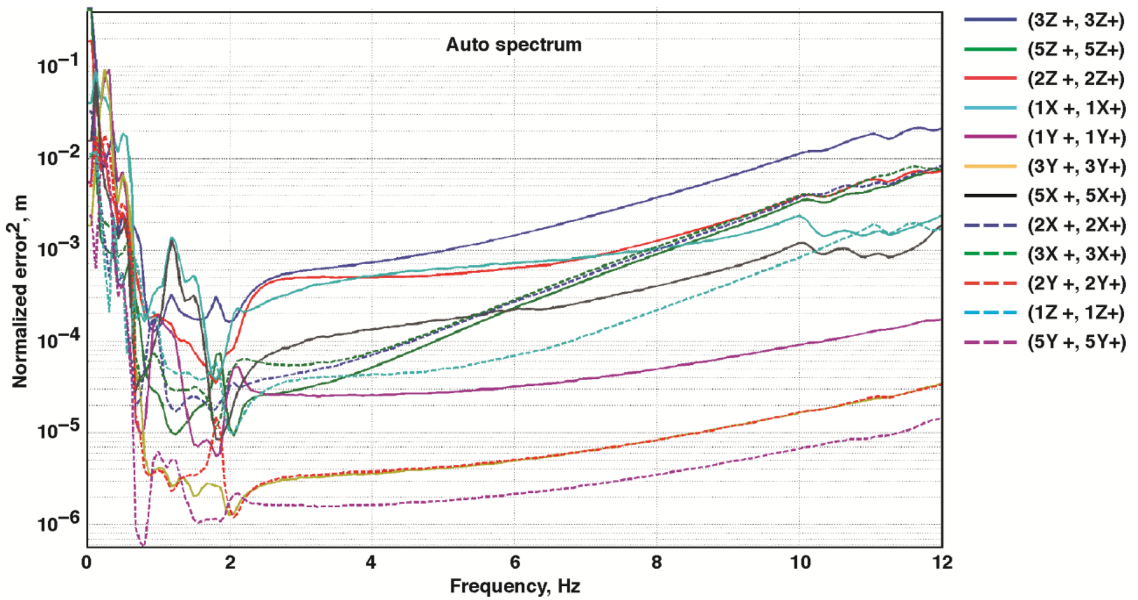


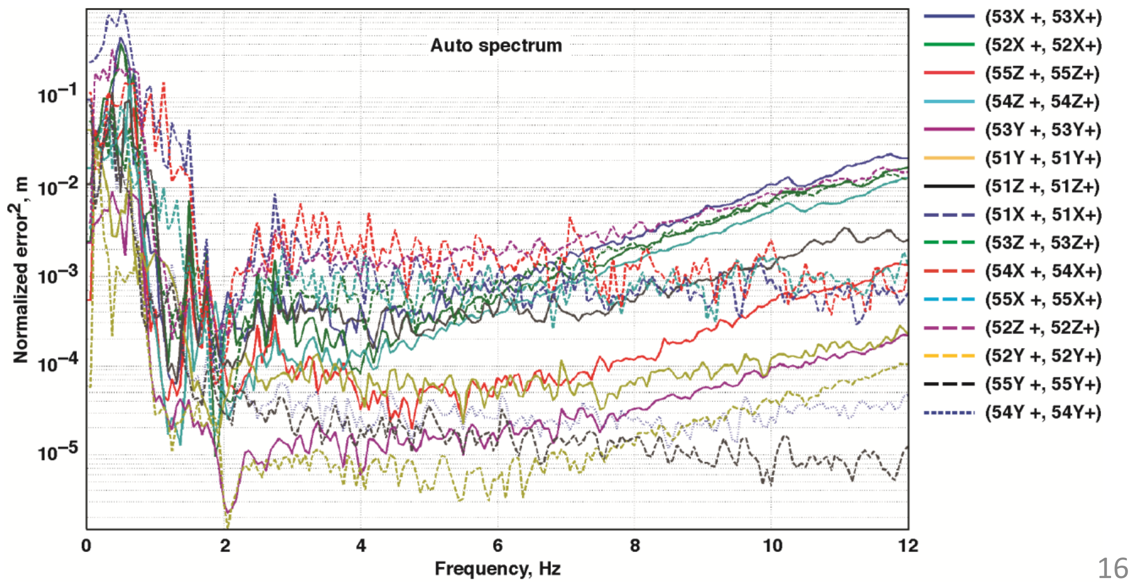
Normalized Error Function

- Normalized error function provides a method of outlining outlier accels

- Seismic accels provided cleaner data.

- Typical GVT accels provided noisier data.





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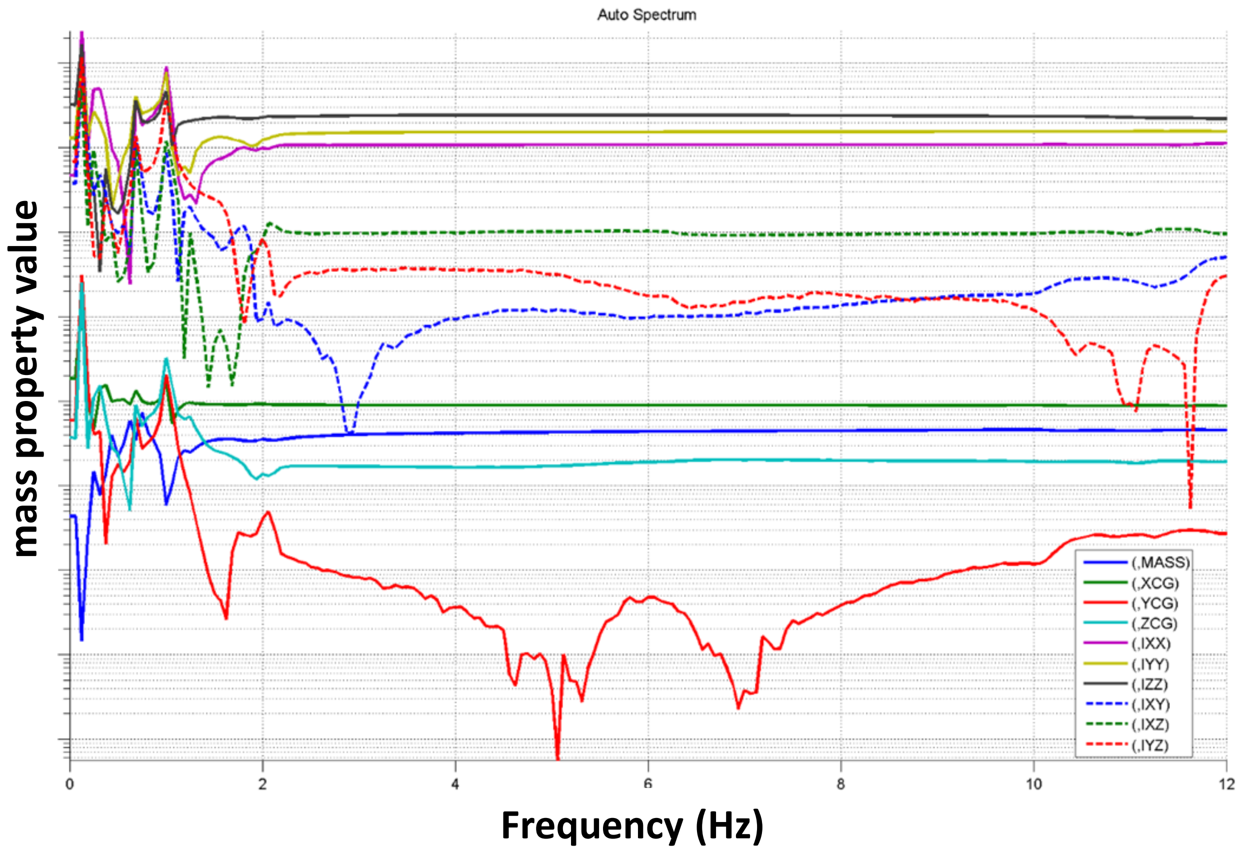
DIM Results

- The computed mass, MOI, POI, and CG values are plotted as a function of frequency for DIM analysis case 1 in Fig. 11 for a 2- to 12-Hz DIM analysis. The mass, XCG and ZCG, three MOIs, and Ixz functions are relatively flat from 2 Hz to 12 Hz. The YCG, Ixy, and Iyz functions exhibit greater fluctuations, but since these values are nominally zero and the estimated values are very small compared to the other CG and POI values, these fluctuations are to be expected.



DIM Results

- Several parameters of the DIM analyses were varied to investigate the effects on the results.
- Off-axis forces
- Frequency range

Analysis case	MASS, kg	Mass, % error	X_{CG}, m	X_{CG}, % error	Y_{CG}, m	Z_{CG}, m	Z_{CG}, % error	I_{XX}, kg*m²	I_{XX}, % error	I_{YY}, kg*m²	I_{YY}, % error	I_{ZZ}, kg*m²	I_{ZZ}, % error	I_{XY}, kg*m²	I_{XZ}, kg*m²	I_{XZ}, % error	I_{YZ}, kg*m²
	8-to 10-Hz frequency range																
1	7,981	3.40%	2.273	-0.60%	-0.0178	0.485	2.00%	12349.4	-2.70%	17616.9	2.30%	26952.1	-5.50%	-172.7	-1106.2	-21.00%	155.1
1 (no off-axis force)	7,624	-1.20%	2.278	-0.30%	-0.0051	0.462	-2.70%	12290.9	-3.30%	17470.6	1.50%	26981.4	-5.30%	-117.1	-1123.7	-19.80%	149.2
7	7,568	-1.90%	2.271	-0.70%	0.0737	0.259	-45.30%	8896.3	-30.00%	17060.9	-0.80%	22240.6	-22.00%	-1144.2	2838.6	-302.80%	-1267.1
7 (no off-axis force)	7,214	-6.50%	2.283	-0.10%	0.0432	0.320	-32.50%	9949.8	-21.70%	17441.3	1.40%	24815.9	-12.90%	-468.2	1387.1	-199.20%	-5.9

Analysis case	MASS,, kg	Mass, % error	X_{CG}, m	X_{CG}, % error	Y_{CG}, m	Z_{CG}, m	Z_{CG}, % error	I_{XX}, kg*m²	I_{XX}, % error	I_{YY}, kg*m²	I_{YY}, % error	I_{ZZ}, kg*m²	I_{ZZ}, % error	I_{XY}, kg*m²	I_{XZ}, kg*m²	I_{XZ}, % error	I_{YZ}, kg*m²
	2- to 12-Hz frequency range																
1	7,623	-1.20%	2.278	-0.30%	-0.005	0.462	-2.60%	12290.9	-3.20%	17470.6	1.50%	27010.7	-5.20%	-120.0	-1120.8	-19.90%	187.3
2	7,631	-1.10%	2.278	-0.30%	-0.010	0.472	-0.70%	12290.9	-3.20%	17382.8	1.00%	27010.7	-5.20%	-155.1	-1114.9	-20.30%	228.3
3	7,600	-1.50%	2.278	-0.40%	-0.013	0.470	-1.00%	12290.9	-3.30%	17382.8	1.00%	27010.7	-5.20%	-155.1	-1094.5	-21.90%	275.1
4	7,474	-3.10%	2.299	0.60%	0.028	0.445	-6.20%	12203.1	-3.90%	16680.5	-3.10%	27537.4	-3.40%	-90.7	-951.1	-32.10%	-275.1
5	7,523	-2.50%	2.299	0.50%	0.025	0.460	-3.40%	12203.1	-3.90%	16622.0	-3.30%	27537.4	-3.40%	-114.1	-942.3	-32.70%	-245.8
6	7,509	-2.70%	2.299	0.50%	0.023	0.457	-3.70%	12203.1	-3.90%	16651.2	-3.20%	27508.2	-3.50%	-111.2	-924.7	-33.80%	-207.8
7	7,209	-6.60%	2.278	-0.30%	0.064	0.297	-37.70%	9627.9	-24.30%	17207.2	-0.10%	24289.1	-14.80%	-640.9	1630.0	-216.60%	-298.5
8	7,316	-5.20%	2.271	-0.70%	0.066	0.335	-29.40%	10213.1	-19.60%	17265.8	0.30%	23703.8	-16.80%	-693.6	1586.1	-213.40%	-418.5
9	7,119	-7.70%	2.291	0.20%	0.046	0.328	-30.90%	9598.6	-24.40%	17265.8	0.40%	23879.4	-16.20%	-772.6	1808.5	-229.20%	-550.2
10	7,263	-5.90%	2.299	0.60%	0.056	0.353	-25.60%	10096.1	-20.50%	16534.2	-3.80%	26191.3	-8.10%	-421.4	719.9	-151.40%	-485.8
11	6,562	-15.00%	2.296	0.50%	0.038	0.462	-2.40%	11003.3	-13.30%	16651.2	-3.20%	26337.6	-7.60%	-301.4	248.7	-117.80%	-310.2
12	7,146	-7.40%	2.309	1.00%	0.043	0.386	-18.60%	10096.1	-20.50%	16387.8	-4.80%	25547.5	-10.40%	-561.9	860.4	-161.50%	-834.0

Summary of Results

The DIM method yielded results that matched within approximately 5 percent of the analytical iron bird mass, CG, and MOI. The I_{xz} POI did not match as well, having with errors exceeding 20 percent, however, the DIM I_{xz} results were still better than the 98-percent error from the pendulum-based testing results due to test setup limitations (that is, shallow tilt angle).

Mass, kg	Mass, error	X_{CG}, m	X_{CG}, % error	Y_{CG}, m	Y_{CG}, % error	Z_{CG}, m	Z_{CG}, % error	I_{XX}, kg*m^2	I_{XX}, % error	I_{YY}, kg*m^2	I_{YY}, % error	I_{ZZ}, kg*m^2	I_{ZZ}, % error	I_{XZ}, kg*m^2	I_{XZ}, % error
Analytical															
7716.513	-	2.286	-	0	-	0.475	-	12700.6	-	17207.2	-	28503.1	-	-1398.8	-
Pendulum swing															
7698.823	-0.2%	2.287	0%	-0.03	-	0.475	0%	12261.6	-3.5%	17938.8	4.3%	27859.3	-2.3%	-2768.4	-97.9%
Dynamic Inertia Measurement															
7622.620	-1.2%	2.278	-0.3%	-0.01	-	0.462	-2.6%	12290.9	-3.2%	17470.6	1.5%	27010.7	-5.2%	-1120.8	19.9%

Recommendations and Considerations

- The Dynamic Inertia Measurement (DIM) method is not yet a fully mature technology for large aerospace vehicles.
- Re-test with iron-bird supported by bladder-type air springs instead of 60K3S.
- Different ways of processing data.
- Method for selecting “flat spot”
- Uncertainty methods
- Spatial filtering
- Better understand the effects of force path
- Investigate the influence of gravity at low frequency.
- Evaluate the effects of off-axis reaction forces.
- Investigate required redundancy for the excitation of the six rigid body modes.

Conclusions

- The Dynamic Inertia Measurement (DIM) method shows promise for mass properties testing applications involving large aerospace vehicles. There were sources of error that required mitigation; for example, the soft-support system introduced modes into the test data.
- The next step in the maturation of the DIM method would be to apply the technique to a full-scale aerospace vehicle.
- The three soft support configuration allowed a maximum 2- to 12-Hz frequency band from which to estimate inertia and center of gravity values.
- Single-shaker configurations provided the best results.
- The DIM mass properties testing method requires expensive sensors and equipment.
- Performing the DIM test can simultaneously provide the same modal characteristics data used for ground vibration testing analysis.
- The DIM method, with further development, may prove to be a more efficient approach to estimating the mass properties of a large aerospace vehicle.

Acknowledgements

- NASA Aeronautics Research Mission Directorate
  - Aerosciences Project
  - Aeronautics Test Program

- Testing support personnel at the Flight Loads Laboratory at Armstrong Flight Research Center

- AFRC Aerospace engineers Bob Clarke and Adam Harding for their support and expertise in performing the conventional pendulum mass properties tests.

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Questions?

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